Math: Question 1

Which of the following is equivalent to $4x+6=12$?

A. $2x+4=6$
B. $x+3=3$
C. $3x+2=4$
D. $2x+3=6$

Choice D is correct. Dividing each side of the original equation by 2 yields $\frac{4x+6}{2}=\frac{12}{2}$, which simplifies to $2x+3=6$.

Choice A is incorrect. Dividing each side of the original equation by 2 gives $2x+3=6$, which is not equivalent to $2x+4=6$. Choice B is incorrect. Dividing each side of the original equation by 4 gives $x+\frac{3}{2}=3$, which is not equivalent to $x+3=3$. Choice C is incorrect. Dividing each side of the original equation by 3 gives $\frac{4}{3}x+2=4$, which is not equivalent to $3x+2=4$.

Question Difficulty: Easy

Math: Question 2

A dance teacher ordered outfits for students for a dance recital. Outfits for boys cost $26, and outfits for girls cost $35. The dance teacher ordered a total of 28 outfits and spent $881. If b represents the number of outfits the dance teacher ordered for boys and g represents the number of outfits the dance teacher ordered for girls, which of the following systems of equations can be solved to find b and g ?

A. $26b + 35g = 28$
$b + g = 881$

B. $26b + 35g = 881$
$b + g = 28$

C. $26g + 35b = 28$
$b + g = 881$

D. $26g + 35b = 881$
$b + g = 28$

Choice B is correct. Outfits for boys cost $26 each and the teacher ordered b outfits for boys, so the teacher spent 26b dollars on outfits for boys. Similarly, outfits for girls cost $35 each and the teacher ordered g outfits for girls, so the teacher spent 35g dollars on outfits for girls. Since the teacher spent a total of $881 on outfits for boys and girls, the equation 26b + 35g = 881 must be true. And since the teacher ordered a total of 28 outfits, the equation b + g = 28 must also be true.

Choice A is incorrect and may result from switching the constraint on the total number of outfits with the constraint on the cost of the outfits. Choice C is incorrect and may result from switching the constraint on the total number of outfits with the constraint on the cost of the outfits, as well as switching the cost of the outfits for boys with the cost of the outfits for girls. Choice D is incorrect and may result from switching the cost of the outfits for boys with the cost of the outfits for girls.

Question Difficulty: Easy

Math: Question 3

The function g is defined as $g(x) = 5x + a$, where a is a constant. If $g(4) = 31$, what is the value of a ?

A. 30
B. 22
C. 11
D. -23

Choice C is correct. Substituting 4 for x in g(x) = 5x + a gives g(4) = 5(4) + a. Since g(4) = 31, the equation g(4) = 5(4) + a simplifies to 31 = 20 + a. It follows that a = 11.

Choices A, B, and D are incorrect and may result from arithmetic errors.

Question Difficulty: Medium

Math: Question 4

An architect designing a roadway plans to use solar-powered glass panels of equal size in the design. The architect estimates a 20-mile-long road will use 30,000 panels equally spaced. At this rate, which of the following equations represents the number of solar panels, P, for a road that is m miles long?

A. $P = 20m$

B. $P = 1{,}500m$

C. $P = 30{,}000m$

D. $P = 20m + 30{,}000$

Choice B is correct. It is given that a total of 30,000 panels are used for a 20-mile-long road and that the panels are equally spaced, so there are 30,000 ÷ 20 = 1,500 panels per mile. The total number of panels needed for m miles is the product of 1,500 panels/mile × m miles, or 1,500m panels.

Choice A is incorrect. This equation multiplies m by the number of miles in the road instead of the number of panels per mile. Choice C is incorrect. This equation multiplies m by the total number of panels instead of the number of panels per mile. Choice D is incorrect. This equation multiplies m by the number of miles and adding the total number of panels instead of multiplying m by the number of panels per mile.

Question Difficulty: Medium

Math: Question 5

What is the slope of the line with equation $3 - y = 6 - 2x$ in the xy-plane?

A. 6
B. 3
C. 2
D. -2

Choice C is correct. One way to find the slope of the line is to rewrite the given equation in slope-intercept form. Subtracting 3 from both sides of the equation gives –y = 3 – 2x. Multiplying both sides of –y = 3 – 2x by –1 and rearranging the terms gives y = 2x – 3. The slope is the coefficient of x, which is 2.

Choice A is incorrect and may result from an arithmetic error. Choice B is incorrect and may result from switching the slope and the y-intercept and from making a sign error. Choice D is incorrect and may result from taking the coefficient of x without putting the given equation in slope-intercept form.

Question Difficulty: Medium

Math: Question 6

The equivalence between the total energy and the mass of an object is predicted by the special theory of relativity and can be expressed by the formula $E = mc^2$, where E is the total energy, m is the mass of the object, and c is the speed of light. Which of the following correctly expresses m in terms of E and c ?

A. $m = \frac{E}{c^2}$

B. $m = \frac{c^2}{E}$

C. $m = Ec^2$

D. $m = E - c^2$

Choice A is correct. Dividing both sides of the equation by c^2 gives $\frac{E}{c^2} = \frac{mc^2}{c^2}$, which can be rewritten as $m = \frac{E}{c^2}$.

Choices B, C, and D are incorrect. These are not equivalent to the original equation and may result from algebraic errors.

Question Difficulty: Medium

Math: Question 7

$r(x) = x^2 + 5x - 4$

$q(x) = 2x^2 - 3x + 4$

The functions r and q are defined above. Which of the following is equivalent to $3r(x) + 6q(x)$?

A. $15x^2 - 3x + 12$

B. $15x^4 - 3x^2 + 12$

C. $3x^2 - 2x$

D. $27x^2 - 18x$

Choice A is correct. Multiplying the terms of r(x) by 3 gives $3r(x) = (3)x^2 + (3)5x - (3)(4)$, which simplifies to $3r(x) = 3x^2 + 15x - 12$. Multiplying the terms of q(x) by 6 gives $6q(x) = (6)2x^2 - (6)3x + (6)4$, which simplifies to $6q(x) = 12x^2 - 18x + 24$. Therefore, $3r(x) + 6q(x) = (3x^2 + 15x - 12) + (12x^2 - 18x + 24)$, and combining like terms gives $3r(x) + 6q(x) = (3 + 12)x^2 + (15 - 18)x + (-12 + 24)$. Simplifying the result gives $3r(x) + 6q(x) = 15x^2 - 3x + 12$.

Choice B is incorrect and may result from adding exponents when combining like terms. Choice C is incorrect. It is the result of r(x) + q(x), not 3r(x) + 6q(x). Choice D is incorrect. It is the result of (3 + 6)(r(x) + q(x)) and making a sign error, not 3r(x) + 6q(x).

Question Difficulty: Medium

Math: Question 8

Which of the following is equivalent to $(1-p)(1+p+p^2+p^3+p^4+p^5+p^6)$?

A. $1-p^8$

B. $1-p^7$

C. $1-p^6$

D. $1-p^5$

Choice B is correct. Multiplying (1 – p) by each term of the polynomial within the second pair of parentheses gives (1 – p)1 = 1 – p; (1 – p)p = p – p^2; (1 – p)p^2 = p^2 – p^3; (1 – p)p^3 = p^3 – p^4; (1 – p)p^4 = p^4 – p^5; (1 – p)p^5 = p^5 – p^6; and (1 – p)p^6 = p^6 – p^7. Adding these seven expressions together and combining like terms gives 1 + (p – p) + (p^2 – p^2) + (p^3 – p^3) + (p^4 – p^4) + (p^5 – p^5) + (p^6 – p^6) – p^7, which can be simplified to 1 – p^7.

Choices A, C, and D are incorrect and may result from incorrectly identifying the highest power of p in the expressions or incorrectly combining like terms.

Question Difficulty: Medium

Math: Question 9

If $\frac{\sqrt{72}-\sqrt{32}}{2}=2^a$, what is the value of a?

A. 2

B. $\frac{1}{2}$

C. $-\frac{1}{2}$

D. $-\frac{3}{2}$

Choice B is correct. To find the value of a, simplify the left-hand side of the equation. Factoring the expression under the radical gives $\sqrt{72}$ = $\sqrt{2\times2\times2\times3\times3}$, which can be rewritten as $\sqrt{2\times2}\times\sqrt{3\times3}\times\sqrt{2}$; this simplifies to 2 × 3 × $\sqrt{2}$ = $6\sqrt{2}$. Similarly, $\sqrt{32}$ = $\sqrt{2\times2\times2\times2\times2}$, which can be rewritten as $\sqrt{2\times2}\times\sqrt{2\times2}\times\sqrt{2}$; this simplifies to 2 × 2 × $\sqrt{2}$ = $4\sqrt{2}$. Thus, the given equation becomes $\frac{6\sqrt{2}-4\sqrt{2}}{2}=2^a$, which can be simplified to $\sqrt{2}=2^a$. By the properties of exponents, $\sqrt{2}=2^{\frac{1}{2}}$. Thus, $a=\frac{1}{2}$.

Choices A, C, and D are incorrect and may result from arithmetic errors or from misapplying the laws of exponents.

Question Difficulty: Medium

Math: Question 10

In the xy-plane, which of the following is true of the circle with equation $(x+0.5)^2+(y-0.5)^2=0.5$ and the line with equation $x+y=0$?

A. The line never intersects the circle.
B. The line is tangent to the circle.
C. The line cuts the circle into two arcs of unequal lengths.
D. The line cuts the circle into two arcs of equal lengths.

Choice D is correct. The equation of the circle is given in standard form, and the center of the circle is (–0.5, 0.5). The line x + y = 0 passes through the point (–0.5, 0.5), since –0.5 + 0.5 = 0. The line thus contains a diameter of the circle and cuts the circle into two arcs of equal lengths.

Choices A, B, and C are incorrect and may result from miscalculating the position of the circle or the line.

Question Difficulty: Hard

Math: Question 11

An object hangs from a spring. The formula $\ell = 30 + 2w$ relates the length ℓ, in centimeters, of the spring to the weight w, in newtons, of the object. Which of the following describes the meaning of the 2 in this context?

A. The length, in centimeters, of the spring with no weight attached
B. The weight, in newtons, of an object that will stretch the spring 30 centimeters
C. The increase in the weight, in newtons, of the object for each one-centimeter increase in the length of the spring
D. The increase in the length, in centimeters, of the spring for each one-newton increase in the weight of the object

Choice D is correct. The value 2 is multiplied by w, the weight of the object. When the weight is 0, the length is 30 + 2(0) = 30 centimeters. If the weight increases by w newtons, the length increases by 2w centimeters, or 2 centimeters for each one-newton increase in weight.

Choice A is incorrect because this describes the value 30. Choice B is incorrect because 30 represents the length of the spring before it has been stretched. Choice C is incorrect because this describes the value w.

Question Difficulty: Hard

Math: Question 12

A certain product costs a company $65 to make. The product is sold by a salesperson who earns a commission that is equal to 20% of the sales price of the product. The profit the company makes for each unit is equal to the sales price minus the combined cost of making the product and the commission. If the sales price of the product is $100, which of the following equations gives the number of units, u, of the product the company sold to make a profit of $6,840 ?

A. $(100(1-0.2)-65)u=6{,}840$

B. $(100-65)(1-0.8)u=6{,}840$

C. $0.8(100)-65u=6{,}840$

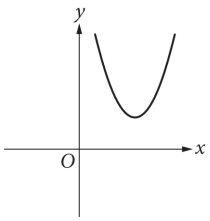
D. $(0.2(100)+65)u=6{,}840$

Choice A is correct. The sales price of one unit of the product is given as $100. Because the salesperson is awarded a commission equal to 20% of the sales price, the expression 100(1 – 0.2) gives the sales price of one unit after the commission is deducted. It is also given that the profit is equal to the sales price minus the combined cost of making the product, or $65, and the commission: 100(1 – 0.2) – 65. Multiplying this expression by u gives the profit of u units: (100(1 – 0.2) – 65)u. Finally, it is given that the profit for u units is $6,840; therefore (100(1 – 0.2) – 65)u = $6,840.

Choice B is incorrect. In this equation, cost is subtracted before commission and the equation gives the commission, not what the company retains after commission. Choice C is incorrect because the number of units is multiplied only by the cost but not by the sale price. Choice D is incorrect because the value 0.2 shows the commission, not what the company retains after commission.

Question Difficulty: Hard

Math: Question 13



The graph of the equation $y = 3x^2 + bx + 5$, where b is a constant, is shown in the xy-plane above. Which of the following could be the value of b ?

A. 9
B. 0
C. -6
D. -15

Choice C is correct. The graph shows that all x- and y-coordinates of the graph are positive. The given equation $y = 3x^2 + bx + 5$ is in standard form, or $y = ax^2 + bx + c$. It is therefore given that $a = 3$. The x-coordinate of the vertex of the parabola can be found using the formula $x = -\frac{b}{2a}$. Since the value of x is positive, the value of b must be negative, and only choices C and D satisfy this condition. Substituting -6 for b into $x = -\frac{b}{2a}$ gives the x-coordinate of the vertex, $x = -\frac{(-6)}{2(3)} = 1$. Substituting 1 for x in the given equation gives the y-coordinate of the vertex: $3(1)^2 - 6(1) + 5 = 2$, which satisfies the condition that the vertex has both positive x- and y-coordinates.

Choices A and B are incorrect because b must be negative in order for the x-coordinate of the vertex of the parabola to be positive. Choice D is incorrect because if b = –15, the given equation would be $y = 3x^2 - 15x + 5$. While the x-coordinate of the vertex, $x = -\frac{(-15)}{2(3)} = \frac{15}{2(3)} = \frac{5}{2}$, is positive, it does not yield a positive y-coordinate of the vertex: $y = 3\left(\frac{5}{2}\right)^2 - 15\left(\frac{5}{2}\right) + 5 = -\frac{55}{4}$.

This does not satisfy the condition that all x- and y-coordinates of the graph are positive.

Question Difficulty: Hard

Math: Question 14

$$y - x^2 + 2 = 9x$$
$$y = 4x + 4$$

If (x, y) is a solution to the system of equations above and $x > 0$, what is the value of x ?

The correct answer is 1. Isolating y in the first equation results in $y = x^2 + 9x - 2$. Since $y = 4x + 4$, the two expressions equal to y can be set equal to each other: $x^2 + 9x - 2 = 4x + 4$. This can be simplified to $0 = x^2 + 5x - 6$ and can be factored as $0 = (x + 6)(x - 1)$. Solving for each value of x yields $x = -6$ and $x = 1$. Since it is given that $x > 0$, 1 must be the correct answer.

Question Difficulty: Hard

Math: Question 15

An industrial printer prints a minimum of 40 brochures per minute and a maximum of 50 brochures per minute, depending on the complexity of the brochure design. What is one possible number of minutes the printer will take to print 3200 brochures?

The correct answer is any number greater than or equal to 64 and less than or equal to 80. The total number of copies, 3200, should be divided by the rate per minute. If the rate is 50, the printer takes 3200 ÷ 50 = 64 minutes. If the rate is 40, the printer takes 3200 ÷ 40 = 80 minutes. Because these rates are given as the minimum and maximum, any value between 64 and 80 inclusive is a plausible number of minutes for the printer to print 3200 brochures.

Question Difficulty: Medium

Math: Question 16

$$8x - 4y = 7$$
$$5y - 4x = 10$$

What is the value of y in the solution of the system of equations above?

The correct answer is 4.5 or $\frac{9}{2}$. Multiplying both sides of the second equation by 2 results in 10y – 8x = 20, which can be rewritten as –8x + 10y = 20. The equations can then be added together:

$$\begin{array}{r} 8x - 4y = 7 \\ +(-8x + 10y = 20) \\ \hline 6y = 27 \end{array}$$

Dividing both sides of the resulting equation by 6 yields y = 4.5.

Question Difficulty: Hard

Math: Question 17

What is the sum of the solutions to the equation $0 = 2x^2 - 5x - 3$?

The correct answer is 2.5 or $\frac{5}{2}$. The original equation can be factored as 0 = (x – 3)(2x + 1). The solutions can be found by solving 0 = x – 3 and 0 = 2x + 1 using the zero product property. Therefore, the solutions to the equation are x = 3 and x = –0.5. The sum of the solutions is 3 + (–0.5) = 2.5.

Question Difficulty: Hard