Math: Question 1

Which of the following is an equivalent form of the expression $15x + 24ax$?

A. $39ax^2$
B. $39(a + 2x)$
C. $(5 + 8a)x$
D. $(15 + 24a)x$

Choice D is correct. The expression $15x + 24ax$ contains two terms with common factors. One of the common factors is x. Factoring x from the expression gives $x(15 + 24a)$, which can also be written as $(15 + 24a)x$.

Choices A, B, and C are incorrect and may result from incorrectly combining and/or factoring the two terms of the expression. One can check that the expressions in each of these choices are not equivalent to the given expression. For example, in choice A, for $x = 1$ and $a = 0$, the value of the given expression is 15 and the value of the expression $39ax^2$ is 0.

Question Difficulty: Medium

Math: Question 2

The formula d = rt is used to calculate the distance an object travels over a period of time, t, at a constant rate, r. Based on this formula, what is the rate, r, in terms of d and t ?

A. $r = \frac{d}{t}$
B. r = dt
C. $r = \frac{t}{d}$
D. r = d – t

Choice A is correct. Dividing each side of the equation d = rt by t results in an equation that expresses r in terms of the other variables:

$$r = \frac{d}{t}$$

.

Choices B, C, and D are incorrect and may result from algebraic errors when rewriting the given equation.

Question Difficulty: Easy

Math: Question 3

Which of the following ordered pairs (x, y) satisfies both equations $y = x^2 + 3x - 4$ and $x = y - 4$?

A. (0, –4)
B. (2, 6)
C. (3, 14)
D. (5, 9)

Choice B is correct. The equation $x = y - 4$ can be rewritten as $y = x + 4$. Substituting $x + 4$ for y in the other equation gives $x + 4 = x^2 + 3x - 4$, which can be rewritten as $x^2 + 2x - 8 = 0$. Since –4 and 2 are the two numbers whose sum is –2 and whose product is –8, they are the solutions to the equation $x^2 + 2x - 8 = 0$. From the equation $y = x + 4$, it follows that the solutions of the system are (–4, 0) and (2, 6). Therefore, of the given choices, (2, 6) is the correct answer.

Choices A and C are incorrect because each of these ordered pairs satisfies the quadratic equation but not the linear equation. Choice D is incorrect because this ordered pair satisfies the linear equation but not the quadratic equation.

Question Difficulty: Medium

Math: Question 4

Which of the following is a solution to the equation $2x^2 + 4x = 3 + 3x^2$?

A. -1
B. 0
C. 3
D. 6

Choice C is correct. The given equation can be rewritten as $x^2 - 4x + 3 = 0$. Since 1 and 3 are two numbers whose sum is 4 and whose product is 3, it follows that they are the solutions to the equation $x^2 - 4x + 3 = 0$. Therefore, of the choices given, only 3 can be a solution to the original equation.

Choices A, B, and D are incorrect because none of these values satisfy the given equation.

Question Difficulty: Medium

Math: Question 5

$$-3x - 4y = 20$$
$$x - 10y = 16$$

If
(x, y)
is the solution to the system of equations above, what is the value of x ?

A. –14
B. –12
C. –4
D. 16

Choice C is correct. Multiplying each side of the second equation by 3 and then adding the equations eliminates x, as follows:

$$\begin{array}{l} \left\{ \begin{array}{l} -3x - 4y = 20 \\ 3x - 30y = 48 \end{array} \right. \\ \hline \ 0 - 34y = 68 \end{array}$$

Solving the obtained equation for y gives y = –2.
Substituting –2 for y in the second equation of the system gives x – 10(–2) = 16, which simplifies to x + 20 = 16, or x = –4.

Choices A, B, and D are incorrect because there is no solution to the system for which the x-coordinate is one of the numbers given in these choices. For example, substituting –14 for x in the second equation gives y = –3. But the pair (–14, –3) does not satisfy the first equation, and it is therefore not a solution to the system of equations.

Question Difficulty: Medium

Math: Question 6

The equation $y = 36 + 18x$ models the relationship between the height y, in inches, of a typical golden delicious apple tree and the number of years, x, after it was planted. If the equation is graphed in the xy-plane, what is indicated by the y-intercept of the graph?

A. The age, in years, of a typical apple tree when it is planted
B. The height, in inches, of a typical apple tree when it is planted
C. The number of years it takes a typical apple tree to grow
D. The number of inches a typical apple tree grows each year

Choice B is correct. If the equation $y = 36 + 18x$ is graphed in the xy-plane, the y-intercept is at $(0, 36)$. Since y represents the height, in inches, of a typical apple tree and x represents the number of years after it was planted, it follows that the number 36 represents the height, in inches, of a typical apple tree when $x = 0$; that is, the height, in inches, at the time the apple tree is planted.

Choice A is incorrect and may be the result of confusing the age of the tree with its height. Choice C is incorrect because the equation provided does not indicate when a typical apple tree will stop growing. Choice D is incorrect and may be the result of confusing the y-intercept with the slope of the line $y = 36 + 18x$.

Question Difficulty: Medium

Math: Question 7

Giovanni wants to buy shirts that cost \$19.40 each and sweaters that cost \$24.80 each. An 8% sales tax will be applied to the entire purchase. If Giovanni buys 2 shirts, which equation relates the number of sweaters purchased, p, and the total cost in dollars, y ?

A. $1.08(38.80 + 24.80p) = y$
B. $38.80 + 24.80p = 0.92y$
C. $38.80 + 24.80p = 1.08y$
D. $0.92(38.80 + 24.80p) = y$

Choice A is correct. The cost, in dollars, of Giovanni's 2 shirts is $19.40 \times 2 = 38.80$, and the cost, in dollars, of his p sweaters is $24.80 \times p = 24.80p$. Additionally, he paid an 8% sales tax. To include the sales tax in the total cost, the combined cost of shirts and sweaters should be multiplied by 1.08. Therefore, the total cost, in dollars, of Giovanni's purchases, y, can be expressed as $1.08(38.80 + 24.80p)$.

Choice B is incorrect and may result from using the factor $1 - 0.08 = 0.92$, instead of $1 + 0.08 = 1.08$, to calculate the sales tax and from multiplying by this factor on the wrong side of the equation. Choice C is incorrect and may result from multiplying by the sales tax factor on the wrong side of the equation. Choice D is incorrect and may result from using the factor $1 - 0.08 = 0.92$ instead of $1 + 0.08 = 1.08$ to calculate the sales tax.

Question Difficulty: Medium

Math: Question 8

A line is graphed in the xy-plane. If the line has a positive slope and a negative y-intercept, which of the following points cannot lie on the line?

A. (–3, –3)
B. (–3, 3)
C. (3, –3)
D. (3, 3)

Choice B is correct. Any line that passes through the point (–3, 3) and has a positive slope will intersect the y-axis at a point (0, b) with b > 3; that is, such a line will have a y-intercept greater than 3. Therefore, a line that has a positive slope and a negative y-intercept cannot pass through the point (–3, 3).

Choices A, C, and D are incorrect because they are points that a line with a positive slope and a negative y-intercept could pass through. For example, in choice A, the line with equation

$$y = \frac{1}{3}x - 2$$

has a positive slope

$$(\frac{1}{3})$$

and a negative y-intercept (–2) but passes through the point (–3, –3).

Question Difficulty: Medium

Math: Question 9

A parachute design uses 18 separate pieces of rope. Each piece of rope must be at least 270 centimeters and no more than 280 centimeters long. What inequality represents all possible values of the total length of rope x, in centimeters, needed for the parachute?

A. $270 \le x \le 280$
B. $4{,}860 \le x \le 4{,}870$
C. $4{,}860 \le x \le 5{,}040$
D. $5{,}030 \le x \le 5{,}040$

Choice C is correct. If the length, in centimeters, of one piece of rope is represented by q, and each piece of rope must be at least 270 centimeters and no more than 280 centimeters long, then it follows that $270 \le q \le 280$. In turn, the total length x, in centimeters, of rope needed for the parachute is 18q because 18 pieces are needed. So, since $x = 18q$, multiplying all the terms of the inequality $270 \le q \le 280$ by 18 gives $(270 \times 18) \le 18q \le (280 \times 18)$, or $4{,}860 \le x \le 5{,}040$.

Choice A is incorrect and may result from mistakenly using x for the length, in centimeters, of one piece of rope instead of the total length of rope. Choice B is incorrect and may result from multiplying the single-piece lower limit for length by 18 and then adding 10 to create the total upper limit, instead of multiplying both the single-piece lower and upper limits by 18. Choice D is incorrect and may result from multiplying the single-piece upper limit for length by 18 and then subtracting 10 to create the total lower limit, instead of multiplying both the single-piece lower and upper limits by 18.

Question Difficulty: Medium

Math: Question 10

A carpenter has \$60 with which to buy supplies. The carpenter needs to buy both nails and screws. Nails cost \$12.99 per box, and screws cost \$14.99 per box. If n represents the number of boxes of nails and s represents the number of boxes of screws, which of the following systems of inequalities models this situation?

A. $\begin{cases} 12.99n + 14.99s \geq 60 \\ n + s \leq 1 \end{cases}$

B. $\begin{cases} 12.99n + 14.99s \leq 60 \\ n + s \leq 1 \end{cases}$

C. $\begin{cases} 12.99n + 14.99s \geq 60 \\ n \geq 1 \\ s \geq 1 \end{cases}$

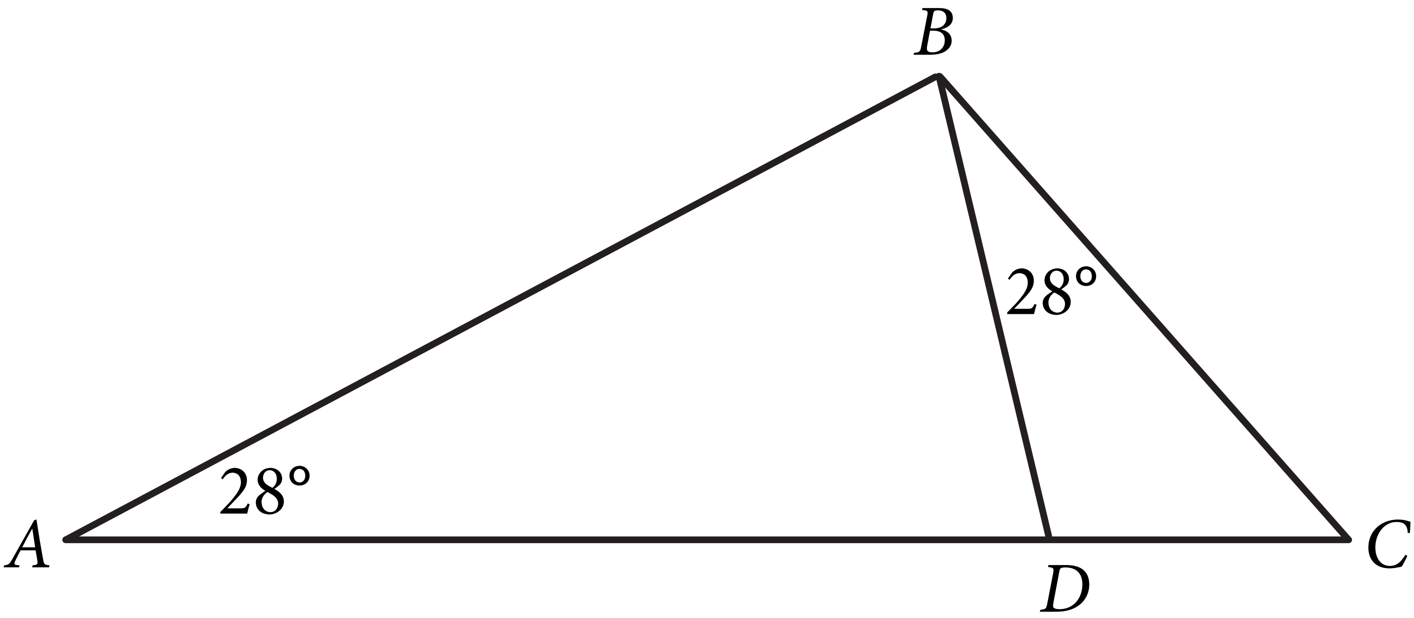
D. $\begin{cases} 12.99n + 14.99s \leq 60 \\ n \geq 1 \\ s \geq 1 \end{cases}$

Choice D is correct. Since the carpenter needs to buy both nails and screws, at least one box of each needs to be purchased. This can be expressed by the pair of inequalities $n \geq 1$ and $s \geq 1$. However, the number of boxes the carpenter can buy is limited by a budget of \$60. The amount, in dollars, the carpenter spends on nails or screws can be expressed as the price of each box multiplied by the number of each type of box, which is $12.99n$ for nails and $14.99s$ for screws. And since this total cannot exceed \$60, it follows that $12.99n + 14.99s \leq 60$.

Choice A is incorrect because the first inequality allows the total cost of nails and screws to exceed the carpenter's budget of \$60, and the second inequality incorrectly expresses the constraint on the number of boxes that the carpenter can buy. That number must be greater than 1, since the carpenter must buy at least one box of nails <u>and</u> one box of screws. Choice B is incorrect because the second equation incorrectly expresses the constraint on the number of boxes that the carpenter can buy. That number must be greater than 1, since the carpenter must buy at least one box of nails <u>and</u> one box of screws. Choice C is incorrect because the first inequality allows for the total cost to exceed the carpenter's budget of \$60.

Question Difficulty: Medium

Math: Question 11



In the figure above, which of the following ratios has the same value as

$$\frac{AB}{BC}$$

?

A. $\frac{BD}{DC}$

B. $\frac{BC}{AC}$

C. $\frac{AD}{BD}$

D. $\frac{DC}{BC}$

Choice A is correct. In the figure, triangles ABC and BDC are similar because each has an angle that measures 28°, and they share angle C. Thus their corresponding sides are in proportion. The sides AB in triangle ABC and BD in triangle BDC correspond to each other because they are opposite the same angle in both triangles (angle C), and the sides BC in triangle ABC and DC in triangle BDC correspond to each other because they are opposite the congruent angles with measure 28° in the corresponding triangles. Therefore,

$$\frac{AB}{BC} = \frac{BD}{DC}$$

.

Choices B, C, and D are incorrect because they are ratios that do not have the same value as

$$\frac{AB}{BC}$$

and are likely the result of misunderstanding which triangles are similar or which sides of the triangles are corresponding sides.

Question Difficulty: Hard

Math: Question 12

$$(x^2y^3)^{\frac{1}{2}}(x^2y^3)^{\frac{1}{3}} = x^{\frac{a}{3}}y^{\frac{a}{2}}$$

If the equation above, where a is a constant, is true for all positive values of x and y, what is the value of a ?

A. 2
B. 3
C. 5
D. 6

Choice C is correct. After distributing the outside exponents to each expression within the parentheses by the rules of exponents, the left side of the equation can be rewritten as

$$(x^2y^3)^{\frac{1}{2}}(x^2y^3)^{\frac{1}{3}} = (x^{(2)(\frac{1}{2})}y^{(3)(\frac{1}{2})})(x^{(2)(\frac{1}{3})}y^{(3)(\frac{1}{3})}) = (xy^{\frac{3}{2}})(x^{\frac{2}{3}}y)$$

. Multiplying the expressions within the parentheses and applying the exponent rules yields

$$x^{1+\frac{2}{3}}y^{\frac{3}{2}+1} = x^{\frac{5}{3}}y^{\frac{5}{2}}$$

, which means the equation

$$x^{\frac{5}{3}}y^{\frac{5}{2}} = x^{\frac{a}{3}}y^{\frac{a}{2}}$$

is true for all positive values of x and y. It follows that the corresponding exponents of x and y on both sides of the equation must be equal, which yields a = 5.

Choices A, B, and D are incorrect and may result from errors when applying the rules of exponents to the given expression.

Question Difficulty: Hard

Math: Question 13

If the equation $y = (x - 6)(x + 12)$ is graphed in the xy-plane, what is the x-coordinate of the parabola's vertex?

A. -6
B. -3
C. 3
D. 6

Choice B is correct. The graph of $y = (x - 6)(x + 12)$ is a parabola that opens upward and has a vertical axis of symmetry. The vertex of the parabola lies on this axis of symmetry, and the x-intercepts of the parabola are equidistant from the axis of symmetry. Since the equation $y = (x - 6)(x + 12)$ is in factored form, the x-intercepts of its graph are $(6, 0)$ and $(-12, 0)$. Therefore, the axis of symmetry is the line

$$x = \frac{6+(-12)}{2}$$

, or $x = -3$. Because the vertex lies on the line $x = -3$, the x-coordinate of the vertex must also be $x = -3$.

Choices A, C, and D are incorrect and may result from misunderstanding the relationship between the given equation and the x-intercepts of the parabola as well as the relationship between the x-intercepts of the parabola and the x-coordinate of the parabola's vertex. For example, choice C may result from mistakenly taking the x-intercepts of the graph of $y = (x - 6)(x + 12)$ as $(-6, 0)$ and $(12, 0)$ instead of as $(6, 0)$ and $(-12, 0)$.

Question Difficulty: Hard

Math: Question 14

$21x + 14 = 7(3x + a)$

In the equation above, a is a constant. For what value of a does the equation have an infinite number of solutions?

The correct answer is 2. If a linear equation is written in the form $mx + n = px + r$, where $m = p$ and $n = r$, then the linear equation is satisfied by any value of x and will have infinitely many solutions. Distributing 7 on the right-hand side of the given equation yields $21x + 14 = 21x + 7a$. Therefore, the equation will have infinitely many solutions if $14 = 7a$; that is, if $a = 2$.

Question Difficulty: Medium

Math: Question 15

Juliene practiced her dance routine for twice as many minutes on Monday as she did on Tuesday. She practiced her routine those two days for a total of 2 hours and 15 minutes. For how many minutes did Juliene practice her dance routine on Monday?

The correct answer is 90. Juliene practiced twice as long on Monday as she did on Tuesday. Therefore, if x is the number of minutes Juliene practiced on Tuesday, then $2x$ is the number of minutes she practiced on Monday. The total amount of time Juliene practiced on the two days is 2 hours and 15 minutes, which is equal to 135 minutes. Thus, the equation $x + 2x = 135$ must be true. This simplifies to $3x = 135$, and so $x = 45$. The number of minutes Juliene practiced on Monday is $2x$, which is equal to $2x = 2(45) = 90$.

Question Difficulty: Hard

Math: Question 16

In the expression below, a is an integer.

$12x^2 + ax - 20$

If $3x + 4$ is a factor of the expression above, what is the value of a ?

The correct answer is 1. It is given that one factor of the quadratic expression is $3x + 4$. Thus, $12x^2 + ax - 20 = (3x + 4)(mx + p)$, where a, m, and p are integers. Multiplying out the right-hand side of the equation gives $12x^2 + ax - 20 = 3mx^2 + (3p + 4m)x + 4p$. It follows that $12 = 3m$, $a = 3p + 4m$, and $-20 = 4p$. Dividing both sides of the equation $12 = 3m$ by 3 gives $m = 4$. Dividing both sides of the equation $-20 = 4p$ by 4 gives $p = -5$. Finally, substituting $m = 4$ and $p = -5$ in the equation $a = 3p + 4m$ gives $a = 3(-5) + 4(4) = 1$.

Question Difficulty: Hard

Math: Question 17

(ax + by)(cx – dy)

In the expression above, a, b, c, and d are non-zero constants and ad = bc. If ac = 18 and bd = 50, what is the value of the coefficient of the xy term when the expression is multiplied out and the like terms are collected?

The correct answer is 0. Multiplying out the given expression yields (ax + by)(cx – dy) = acx^2 + (bc – ad)xy – bdy^2. Since ad = bc, the coefficient of the xy term, bc – ad, is 0.

Question Difficulty: Hard